VISUAL PROCESSING ON GRAPHICS TASK: The case of a street map


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Tracy Logan and Tom Lowrie argue that while little attention is given to visual imagery and spatial reasoning within the Australian Curriculum, a significant proportion of NAPLAN tasks require high levels of visuospatial reasoning. This article includes teaching ideas to promote visuospatial reasoning in the primary classroom.

Introduction

Spatial and visual reasoning are essential ingredients of mathematical thinking and processing (Owens & Outhred, 2006; Presmeg, 2006). Although visual imagery and spatial reasoning are given scant attention in the new Australian Curriculum (Lowrie, Logan & Scriven, 2012), there is certainly a strong emphasis on such reasoning in the national assessment instruments. For example, in the 2012 National Assessment Program Literacy and Numeracy (NAPLAN) Year 3 numeracy instrument, a high proportion of the tasks require students to: mentally rotate three-dimensional (3D) objects and two-dimensional (2D) shapes; navigate maps; visualise number patterns; rotate and reflect objects; and mentally construct and deconstruct 3D objects. In fact, 13 of the 35 tasks (37%) required high levels of visuospatial reasoning.

Visuospatial reasoning is processed in two ways: (1) by evoking mental imagery (usually in the mind's eye) (Kosslyn, 1983); and/or (2) representing images in concrete or dynamic ways (e.g., drawing diagrams) (Diezmann & English, 2001). In an earlier edition of APMC, we argued (Lowrie & Logan, 2007) that visual processing was an essential component of mathematics reasoning and that such processing was particularly important when students encountered spatial tasks such as maps. It is important to foster such skills from a young age, and as teachers we need to ensure that our pedagogical practices

provide opportunities for students to engage with such processing—even though it is not explicitly addressed in the new mathematics curriculum.

Map tasks

There are a variety of map task graphics used in assessment items and indeed in classroom activities that promote visuospatial reasoning. The most common type of map representations include graphics that represent the *location and arrangement of objects* and include street maps, pictorial maps and coordinate maps.

According to Liben (2008), the cognitive skills that relate specifically to maps are *representation* (the content, the what and how of maps) and *space* (spatial information such as scale, direction, and angle). Liben maintained that children encounter difficulties interpreting maps when they: (1) misinterpret the representation of symbols (for example, believing that the symbol represented on the map has the same attributes in the real world); and/or (2) become confused about perspectives and different angles used to represent different maps (for example, elevation view and bird's-eye view). Hence, reading and understanding a map is a skill in itself, with certain fundamental features that need to be taken into consideration.

Research on map tasks

As part of a study that involved over 1000 Year 6 students (from Australia and Singapore) solving mathematics tasks, we identified a number of visuospatial strategies that students utilised both to decode graphics and encode spatial information. In this paper, we report on how 871 of the students solved the Don Road map task (see Figure 1) and explained their mathematics reasoning (by completing a mathematics processing instrument).

A car is travelling **north-east** along Don Road. The car is about to turn right into Plum Road.

In which direction will the car be travelling **after** it turns right?

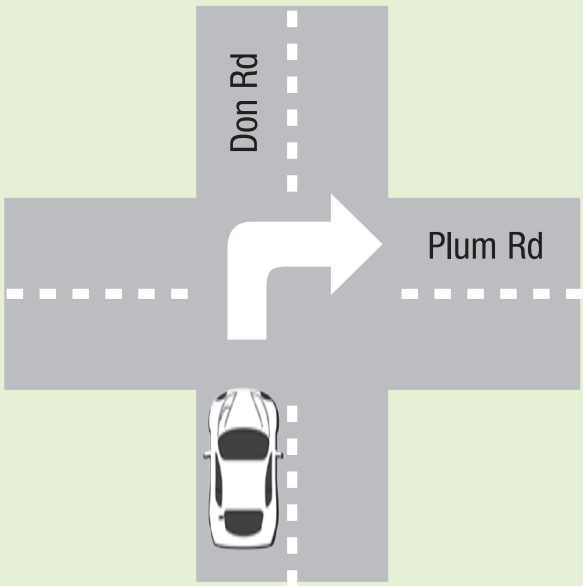

Figure 1. The Don Road task. © Australian Curriculum, Assessment and Reporting Authority, 2010.

The students found this task quite difficult to solve (only 40% solved the task correctly). It required students to orientate the graphic to the north and/or reposition the graphic to the north-east and appreciate that a right turn is a 90° movement of a point (or line) in space. Note that the main challenge of the task involves north not being directed at the top of the page. Successful visuospatial approaches included gesturing, drawing concrete diagrams or utilising dynamic imagery typically evoked through visualisation.

Table 1 provides the results for this task and includes the counts of correct and incorrect responses by processing method.

Table 1. Correct and incorrect responses by processing type.

	Processing (frequency) N = 871			
	Gestural	Concrete/ Pictorial	Visual	Non-visual
Correct	55	267	81	67
Incorrect	80	241	22	58
Total	135	508	103	125

Student processing: One task, many ways to solve it

Pictorial-concrete representations

The majority of students who incorrectly solved the task drew a diagram of a compass but failed to correctly orientate the position of the compass in relation to the task context. This concrete-pictorial representation of a compass (see Figure 2), which contained appropriate bearings, was used to scaffold the student's understandings. The student also appreciated that it was necessary to move 90° in space in order to fulfil the requirement of the car turning right.

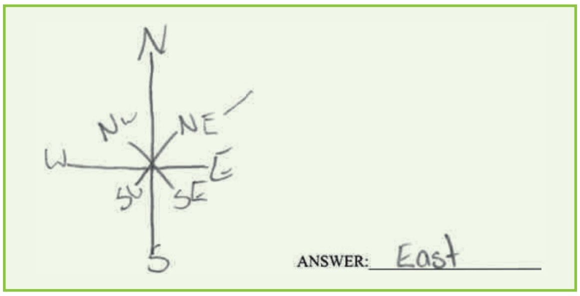

Figure 2. A concrete representation that did not support understanding.

The incorrect solution (i.e., east) was a result of the student producing an encoded image of the compass in a typical representation—with north at the top of the page. This is an understandable error since this is the conventional representation (and orientation) of a compass. These students failed to appreciate that position and movement through space is not fixed—especially given the dynamic scenario presented in the road map task.

In order to utilise effectively the compass bearing, students needed to reorientate the position of the car in relation to the bearing (i.e., ensure that the car was moving north-east). Figure 3 shows a typical illustration done by students who were able to do this. This student drewww the compass accurately, but then highlighted NE on the compass. The student then drew a picture that represented the car moving in the north-east direction and turning right. This diagram was additionally supportive since it identified movement through 90 degrees. The final part of the encoding was to circle the south-east point, indicating this movement.

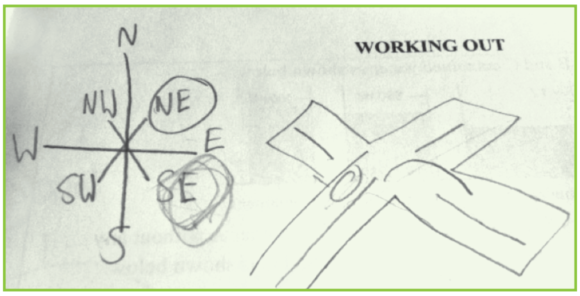

Figure 3. A supportive (and appropriate) use of concrete representations to solve the task.

Other students were able to draw accurately the compass points with north bearing to the top left of the page, so that the car was travelling north-east. Figure 4 highlights this type of concrete/pictorial representation.

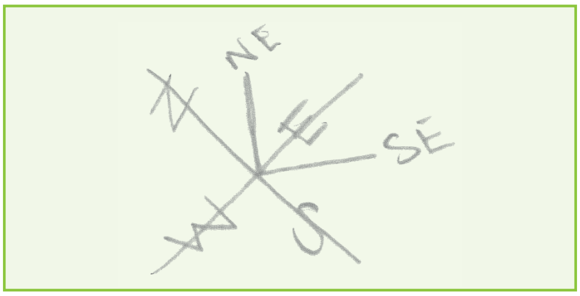

Figure 4. An example of re-aligning the compass points to match the direction the car was travelling.

Gestural representations

Rather than drawing diagrams, some students used gestural behaviours to solve the task. These behaviours included deictic gesturing (McNeill, 1992) which involves pointing movements (usually with fingers or

hands) directed towards objects or events. Approximately 12% of all responses utilised such an approach. Some of these students reported turning the page of their booklet in order to re-orientate the space (see Figure 5). Other students reported using their hands or fingers to pinpoint where north would be on the compass or used gesturing to indicate the movement in space.

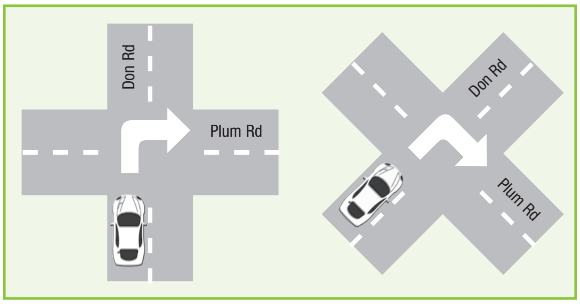

Figure 5. A representation of students turning the page of the booklet to re-orientate the task.

Visualisation

The other main visual process used by students to solve the task involved visualisation. In such situations, the problem solver evokes or generates images "in the mind's eye" (Kosslyn, 1983) to help scaffold understanding. The students reported visualising or imagining where the compass indicating north would be on the map and worked out in which direction the car would be travelling. In this study, 79% of the students who utilised a visualisation process did so correctly. Hence, they were able to visualise the compass point with north-east to the top of the page and identify a 90° turn in their mind. There are, however, limitations to such processing if the problem solver only utilises the 'textbook' orientation of a compass with north bearing straight to the top of the page (similar to the drawing in Figure 2). This can result in an incorrect visual image being produced and therefore an incorrect answer.

Enhancing students visuospatial thinking

This research project highlighted the importance of visuospatial thinking when solving tasks that required high levels of spatial reasoning. More than 85% of the students' responses involved a visual process which supported the students' thinking and scaffold their understanding of the spatial demands of the task. The most frequently used strategies involved visualisation or pictorial-concrete representations of the tasks—which were used in conjunction with the map graphic embedded in the task in an attempt to realign the spatial dimensions of the task (in this case, the rotation of an object or the movement of an object in space). Gestural behaviours were used less frequently but were also an effective way of monitoring the tasks' spatial demands. In the following section, we provide some teaching ideas that can help enhance students' visual reasoning and awareness of utilising such strategies during the problem-solving process.

Teaching ideas for graphic tasks

The importance of graphics cannot be underestimated in the mathematics curriculum. Students need opportunities to develop their understandings about graphics in various mathematical situations. Below are a number of suggestions to help students develop these understandings.

Deconstructing the task

Graphics can be classified in terms of images or objects that: (a) are essential for task solution (information graphics); or (b) provide context but are not required in order to solve the task (contextual graphics; see Diezmann, Lowrie, Sugars & Logan, 2009). In this sense, students need to determine whether the graphic is necessary to solve the task or not. They need to identify whether the intent of the graphic is to provide a context (illustration) or present mathematical information. If the graphic is necessary, students need to consider which information within the graphic is essential (since some information may not be pertinent for the specific question). Another important aspect is for students to be taught the relationships between the graphic and the textual information represented in

the task. Within many graphics tasks, the mathematics content is presented not only within the graphic but also within the text stimulus (as for the map task represented in this paper) and it is imperative that students are encouraged to look carefully at the mathematics information embedded within the tasks (Lowrie, Diezmann, & Logan, 2012). Given that so many students in our study used a stereotypical orientation of a compass, it is beneficial for students to be able to work with uncommon representations and have opportunities to experience visually diverse examples of not only the same type of information graphic, but also a variety of graphics which are used within mathematics.

Encouraging drawing

In many mathematics classrooms today there is a concerted push to teach mental arithmetic and for students to 'work out' their answers using mental strategies. For some students, the use of a more concrete representation is often needed. The use of a self-drawn diagram can help alleviate some of the cognitive burden of students as they solve mathematics problems because drawings can allow students to monitor their thinking and serve to scaffold information. As seen in this study, more than half of the students drew a diagram to scaffold their understanding of the Don Road map task, despite a graphic representation being embedded in the task. Teachers can help students to make the most of such strategies by explicitly teaching about different diagrams that can represent information, for example, networks, matrices, hierarchies, and part–whole diagrams (see Diezmann & English, 2001).

Teachers can also encourage students to represent the information in the task in a way that is meaningful to them and then share this with other students. Students will often represent the task in very different ways with some students providing elaborate drawings while others will use dots and marks as placeholders. It is important, however, to ensure students are aware of the fact that detailed diagrams can often be distracting and actually cause confusion, not to mention can be time consuming to draw. Drawings can also be used to best effect when students are checking solutions or need to have a concrete representation of their working.

Visualisation techniques

Teachers will often use hands-on or digital materials to help students 'see' the mathematics within tasks. What occurs less in mathematics classrooms is the explicit teaching of visualisation techniques with specific reference to mathematics content and processes. Visualisation is an important aspect of mathematical thinking (Presmeg, 2006); teaching and using visualisation techniques in the mathematics classroom have the potential to aid and facilitate mathematical thought. Some teaching ideas include:

- Asking a student to imagine/visualise something very familiar to them and invite that child to describe what it looks, sounds, feels like, etc., in order for other children to start visualising a similar thing. Ask each student to draw the image that they pictured in their mind.
- Have students draw and follow maps of familiar and unfamiliar environments, then remember and visualise in their head the route they followed.
- Using a 'touch and feel box' where students place their hands inside a covered box and using their sense of touch, describe what is inside. For example, inside could be a two- or three-dimensional shape or object and the child has to explain to the class or their partner what attributes it has. The other child/children then have to visualise the shape or object based on the description provided;
- Similarly, in pairs, students sit facing each other with something blocking their view of the other person (e.g., a piece of cardboard). One student makes a pattern or an object out of materials and then has to describe to their partner what it looks like. The other child is encouraged to visualise first, then draw or make the pattern or object.

Concluding thoughts

There is a need to continue research into how students process graphically-rich mathematics tasks. As can be seen from this article, even a task based around one mathematics concept can be (and will be) solved by students in numerous ways. This study involved a large number of children from different contexts and yet the overwhelming outcome shows that students, even at Grade 6, rely on visuospatial approaches when solving graphic tasks. Teachers should encourage students to talk about the visual approaches they use to solve graphic mathematics tasks in the classroom. However, students should be encouraged to rely less on visual representations as they become competent within a domain.

Despite the Australian Curriculum not providing explicit reference to visuospatial reasoning, it is apparent that students utilise such reasoning and processing when working out mathematics tasks, and teachers need to be mindful of how students process these types of tasks.

Note

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